ENSLAPP-A-510/95, hep-ph/9503244

Finite temperature formalism for nonabelian gauge theories in the physical phase space

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(February 1, 2008)

Abstract

We establish a new framework of finite temperature field theory for Yang-Mills theories in the physical phase space eliminating all unphysical degrees of freedoms. Relating our method to the imaginary time formalism of James and Landshoff in temporal axial gauge, we calculate the two-loop pressure and provide a systematic and unique method to construct the additional vertices encountered in their approach.

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INTRODUCTION

This paper is devoted to establish finite temperature field theory on the physical phase space of nonabelian gauge theories. It is the intrinsic nature of every gauge theory that the whole configuration space contains gauge group orbits, and gauge transformations generate shifts along those orbits. Gauge equivalent field configurations are physically indistinguishable, therefore only transitions between distinct gauge orbits contain physical information.

In standard field theory the problem of superficial degrees of freedom is attacked by the introduction of a gauge fixing condition. However, due to Gribov's ambiguity [1], the gauge orbit space of Yang-Mills potentials cannot be parametrized uniquely by potentials satisfying a local gauge condition. A gauge condition surface in the entire configuration space contains gauge equivalent field configurations.

Moreover, there are may exist certain field configurations where the gauge fixing surface is tangential to gauge orbits, corresponding to the zeros of the Faddeev-Popov determinant. Perturbatively, this entails, depending on the gauge chosen, unphysical poles in the propagator that have to be defined properly. In particular for the class of axial gauges, $n \cdot A = 0, \quad n^\mu = (1, \vec{n})$, the spurious poles at $p_0 = \vec{p}\vec{n}$ have to be treated with the co-called Leibbrandt-Mandelstam [2] prescription, which has to be modified [3] for n^μ lightlike. It should be mentioned that Landshoff's α-prescription [4] also gives the correct exponentiation in a Wilson-loop calculation up to order g^4.

At finite temperature in the real time formalism (RTF), analogously to the zero temperature case, a temperature-dependent pole-prescription arises naturally within the framework of Hamiltonian quantization [5], and for the particular choice of temporal axial gauge (TAG), $(\vec{n} = 0)$, a RTF has been developed successfully [6].

In the so-called imaginary time formalism (ITF), however, the energy can only take on the discrete Matsubara frequencies $p_0 = 2\pi i n T$. Thus if one naively heats the unphysical degrees of freedom, an unresolved pole remains for the zero mode for momenta $\vec{p}\vec{n} = 0$. Luckily, in the particular case of the unresummed imaginary part of the transversal structure function of

the gluon self-energy, these factors cancel out for symmetry reasons and the straight-forward application of ITF Feynman rules works [7].

At the contrary, in TAG the naive ITF propagator contains unregularized singular factors $1/p_0$ at zero Matsubara frequency. This problem has been circumvented in earlier works [8] by the ad hoc assumption that such poles have to be dropped. Although there exists no justification for doing so, the leading order self-energy is found to coincide with the results in other gauges. The deeper reason for this is, however, not the correctness of this prescription. In fact, in general axial gauge, the dependence on the gauge fixing vector $\vec{n}$, and thus the axial poles, completely cancels out algebraically [7] which gives the proof that no prescription enters at that loop level. In fact, a closer inspection of the corresponding expressions in TAG reveals that this cancellation takes place in that gauge too. Moreover, this must be the case since the leading-order expression for the self-energy is nothing but the hard thermal loop, and thus a physical, gauge independent quantity.

It is well known, that the consistent calculation of the next-to-leading order contributions requires an appropriate resummation of propagators and vertices [9]. However, since the two-point function is a gauge independent quantity only on the mass-shell, one cannot expect the self-energy to be prescription independent off the physical dispersion relation. In the light of this line of arguments, the non-Debye screening behavior [10,11] which contradicts results obtained in Coulomb gauge [12], and using Polyakov-loop correlators [12,13] appears to be rather an artifact of the off-shell calculation and not of the ad hoc pole prescription. Adopting the on-shell definition of the Debye-mass proposed by Rebhan [14] we expect that in a pragmatic calculation keeping the undefined $1/p_0$ quantities those unphysical poles cancel out.

Taking seriously the naive formulation, it was proposed quite recently [15] to regularize the divergent $1/p_0$ expressions by a temperature dependent expression. However, this proposal may only serve to give the expressions an intermediate meaning, and it has to turn out irrelevant in the calculation of physical quantities. Moreover, it is not clear if this prescription leads a truly temporal propagator and can be adopted unambiguously without

introducing ghost fields. Independent of any ad hoc method to get rid of the temporal pole, there is no justification for the naive application of ITF Feynman rules in TAG.

It has been pointed out by James and Landshoff [16] already some time ago that the temporal pole is related to the free motion of the longitudinal modes of the gauge field. This violates periodic boundary conditions that are necessary to set up ITF. James and Landshoff invented a new formalism in which the longitudinal fields remain unheated and only the remaining physical degrees of freedom attain a temperature. Within this formalism, the longitudinal part of the propagator is automatically free from the $1/p_0$ singularity. It was argued that one can obtain the same answer for the two-loop pressure as in other gauges. The main drawback in that formulation is, however, that one has to construct physical states by explicitly solving the Gauss law which gives rise to additional time-independent vertices. This results in unwieldy expressions and it appears difficult to establish a resummation program.

In the present paper, we advocate a different route to attack the problem. Based on a Hamiltonian formulation of the theory, we are able to eliminate all unphysical degrees of freedoms from the Hamiltonian by introducing an appropriate coordinate system in the space of field configurations which allows to make a unique distinction between gauge degrees of freedoms and physical ones [17]. Within this approach one neither encounters any unphysical poles nor any explicit construction of physical states is necessary. It is straightforward to heat the physical degrees of freedom in the resulting non-local Hamiltonian. In order to illustrate the new method, we calculate the two-loop pressure and compare the result with the corresponding expressions found using the approach of James and Landshoff. Rewriting the physical fields in the basis used in their investigation, we give a general strategy to construct the corresponding Gauss law states to arbitrary order and calculate them explicitly to third order in the coupling constant.

I. CONSTRUCTION OF THE HAMILTONIAN IN THE PHYSICAL PHASE SPACE

In this section we construct the physical Hamiltonian of pure QCD by eliminating all gauge degrees of freedom. The basic idea may be illustrated as follows. Consider a point particle moving in a plane and rotations around the origin as symmetry group. Then different trajectories are gauge equivalent if they can be mapped one to another by rotations. Of course, the dynamics appears simplest in polar coordinates, where only the radial coordinate has a physical meaning. The angular momentum generates gauge transformations and the corresponding canonical conjugate position coordinate, the angle, may be fixed arbitrarily.

We start our investigation with the pure QCD Lagrangian

$$L = -\frac{1}{2}\int d^3x \,\mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu} = -\frac{1}{2}\left\langle F_{\mu\nu}, F^{\mu\nu}\right\rangle \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\left[A_\mu, A_\nu\right]$, and the Yang-Mills fields are elements of the Lie algebra of the gauge group[1]. To go over to the Hamiltonian formalism, we have to determine the canonical momenta $E^\mu = \delta L/\delta \dot{A}^\mu = F^{0\mu}$. The momentum conjugated to A_0 vanishes, $E_0 \sim 0$ forming the primary constraint. The corresponding canonical Hamiltonian has the form $H = 2\left\langle \dot{A}^i, E^i\right\rangle - L = \left\langle E^i, E^i\right\rangle + V(A^i) - 2\left\langle A_0, \mathcal{G}\right\rangle$ where

$$V(A) = \frac{1}{2}\left\langle F^{ij}, F^{ij}\right\rangle \tag{2}$$

appears as potential and $\mathcal{G} = \nabla^i[A]\; E^i = \partial^i E^i - ig[A^i, E^i]$ is the so-called Gauss law, $\nabla^i[A]$ being the covariant derivative in the adjoint representation.

The primary constraint must be conserved during time evolution. This yields the secondary constraints $\dot{E}_0 = \{E_0, H\} = \mathcal{G} \sim 0$, where we implicitly assumed the standard equal-time Poisson brackets $\left\{A^a_\mu(x), E^b_\nu(y)\right\}\Big|_{x_0=y_0} = \delta^{ab} g_{\mu\nu}\delta^3(x-y)$.

Since the algebra of the Gauss law closes, $\left\{\mathcal{G}^a(x), \mathcal{G}^b(y)\right\} = g f^{abc}\delta^3(x-y)\mathcal{G}^c(x)$ and $\dot{\mathcal{G}}^a = \{\mathcal{G}^a, H\} = -f^{abc}A^b_0\mathcal{G}^c$ we conclude that there are no more constraints, and all constraints

[1]The hermitian generators of the gauge group are normalized according to $\mathrm{Tr}\, T^a T^b = \delta^{ab}/2$.

are of the first class. Following the standard Dirac quantization procedure [19], the Poisson brackets may be deformed to eliminate the constraints $E_0 \sim 0$ at the operator level. Since the gauge orbits generated by E_0 are shifts of A_0 only, and leave the other phase space variables A^i, E^μ untouched, we select the gauge equivalent configuration in the phase space which satisfies $A_0 = 0$. This amounts to simply dropping the canonical pair $A_0,\ E_0$ from the Hamiltonian. On this hyperplane, the remaining constraints $\mathcal{G}$ become time independent since the Hamiltonian now commutes with the Gauss law. Those constraints generate time independent gauge transformations on the remaining phase space variables E^i and A^i,

$$E^\Omega = \Omega E \Omega^{-1}, \qquad A^\Omega = \Omega A \Omega^{-1} - \frac{i}{g}(\partial\Omega)\Omega^{-1},$$

where Ω is an element of the gauge group. It is convenient to formulate the quantized theory in a functional representation. Representing the canonical momenta by the standard functional differential operator $E(x) \to -i\delta/\delta A(x)$ the Schrödinger equation takes the form

$$H\Psi_n[A] = \left(-\left\langle \frac{\delta}{\delta A}, \frac{\delta}{\delta A} \right\rangle + V[A] \right) \Psi_n[A] = E_n \Psi_n[A] \tag{3}$$

and the wave function is subject to the constraints

$$\mathcal{G}\Psi_n[A] = \nabla[A]\frac{\delta}{i\delta A}\Psi_n[A] = 0. \tag{4}$$

In the approach of James and Landshoff [16] the wave functions are constructed explicitly by solving this constraint in order to be able to perform the thermal trace over physical states. Alternatively, one may eliminate the superficial degrees of freedom by reducing the number of field components. In analogy to the example given in the beginning of the section, we parametrize the field configurations in the unconstraint configuration space by an 'angle' ω and the remaining coordinates $\hat{A}^i$ in the following way

$$A = U\hat{A}U^{-1} - \frac{i}{g}(\partial U)U^{-1}. \tag{5}$$

Here $U[\omega] = \exp(ig\omega)$ is an element of the gauge group generated by the Lie-algebra valued angle ω and $\hat{A}^i = \epsilon^i_\alpha A^\alpha$, $\alpha = 1, 2$ are the remaining coordinates projected out by the operator

ϵ that is normalized such that $\epsilon^i_\alpha \epsilon^i_\beta = \delta_{\alpha\beta}$ acts as unity in the corresponding subspace. The constraint (4) transforms into

$$\mathcal{G}\ \Psi[\hat{A},\omega] = U\frac{\delta}{\delta i\omega}\Psi[\hat{A},\omega]U^{-1} = 0 \tag{6}$$

which tells us that physical variables are independent of the angle ω. The Gauss law itself turns out to be the canonical conjugate momentum to the 'position' variable ω.

In the potential part $V[A]$ we may simply replace the original gauge field by its physical components, $V[A] = V[\hat{A}]$, since it is gauge invariant under the transformation (5) . For the kinetic part in the Hamiltonian, we need an expression for metric entering implicitly in the definition of the inner product in (1). The lower metric components can be read off from the differentials, (The anihermitian covariant derivative is meant to be taken with respect to the fields $\hat{A}$.)

$$\langle \delta A, \delta A\rangle = \left\langle \delta\hat{A}^\dagger, \delta\hat{A}\right\rangle + \left\langle \delta\omega\nabla^\dagger, \delta\hat{A}\right\rangle + \left\langle \delta\hat{A}^\dagger, \nabla\delta\omega\right\rangle + \left\langle \delta\omega, \nabla^\dagger\nabla\delta\omega\right\rangle$$

i.e. $g_{\alpha\beta} = {\epsilon^i_\alpha}^\dagger \epsilon^i_\beta = \delta_{\alpha\beta}$, $g_{3\alpha} = g^\dagger_{\alpha 3} = \hat{\nabla}_\alpha$, $g_{33} = \nabla^{i\dagger}\nabla^i$, and we defined the projected covariant derivative as $\hat{\nabla}_\alpha = \nabla^i \epsilon^i_\alpha$. We shall also need the determinant of the metric, $\mu^2 = \det(g) = \det(\mathcal{D})$, $\mathcal{D} = \nabla^\dagger\nabla - \hat{\nabla}^\dagger\hat{\nabla}$, and the inverse components, which read $g^{\alpha\beta} = \delta^{\alpha\beta} + \hat{\nabla}^{\alpha\dagger}\mathcal{D}^{-1}\hat{\nabla}^\beta$, $g^{\alpha 3} = g^{3\alpha\dagger} = -\hat{\nabla}^{\alpha\dagger}\mathcal{D}^{-1}$, $g^{33} = \mathcal{D}^{-1}$. The kinetic term in the Hamiltonian reads in covariant form $\langle \delta/\delta X^i, \delta/\delta X^i\rangle = \left\langle \mu^{-1}\delta/\delta X^A\, g^{AB}, \mu\, \delta/\delta X^B\right\rangle$ where we have put $X^A = (A^\alpha, \omega)$. When this operator acts on physical wave functions which by virtue of (6) do not depend on the angle ω, the terms containing the momenta $\delta/\delta\omega$ vanish. Thus the physical Hamiltonian is obtained by simply dropping the 3-components in the metric and the Schrödinger equation (3) together with the constraint (4) is equivalent to the reduced dynamical system described by the Hamiltonian

$$H^{phys} = -\left\langle \mu^{-1}\left(\frac{\delta}{\delta A^\alpha}\right)^\dagger, (\delta^{\alpha\beta} + \hat{\nabla}^{\alpha\dagger}\mathcal{D}^{-1}\hat{\nabla}^\beta)\mu\frac{\delta}{\delta A^\beta}\right\rangle + V[\hat{A}] \tag{7}$$

where the wave function as well as observables are functionals of the coordinates $\hat{A}$ only. Apart from the determinant μ appearing in the Hamiltonian, this expression was already

found in [20] using a different construction. The determinant resolves the apparently existing operator ordering problem which is due to the necessary inversion of the operator $\mathcal{D}$ in the kinetic term. This problem was also discussed previously [21], however without definite solution. The operator μ gives contributions $\sim (\delta^3(0))^2$, which may be dropped if one is only interested in the local properties of the theory.

II. FEYNMAN RULES IN THE PHYSICAL SUBSPACE

The Hamiltonian derived in the previous section may serve to establish a set of Feynman rules in the physical subspace. Since it no more contains unphysical degrees of freedoms, it is straightforward to establish finite temperature field theory by heating the field $\hat{A}$. We emphasize that no explicit choice of the projection operator ϵ^i_α corresponding to particular coordinates is necessary so far. However, (7) contains a non-local operator which makes the theory unwieldy to deal with. Alternatively, one may introduce an auxiliary field and rewrite the Hamiltonian density in the following manner

$$\mathcal{H} = \frac{1}{2}E^{\alpha,a\dagger}E^{\alpha,a} + \frac{1}{2}\left[\Phi^{a\dagger}(\hat{\nabla}^\alpha E^\alpha)^a + (\hat{\nabla}^\alpha E^\alpha)^{a\dagger}\Phi^a\right] - \frac{1}{2}\Phi^{a\dagger}\mathcal{D}^{ab}\Phi^b + \frac{1}{4}\hat{F}^{ij,a}\hat{F}^{ij,a} + \frac{1}{2}\rho^{a\dagger}\mathcal{D}^{ab}\rho^b. \tag{8}$$

The electric field $\hat{E}$ is the canonical conjugate to $\hat{A}$ in operator representation, $[E(x)^{\alpha,a}, A(y)^{\beta,b}]\Big|_{x_0=y_0} = -i\delta^{ab}\delta^{\alpha\beta}\delta^3(x-y)$. The last term in (8) just subtracts off the trace of the operator $\mathcal{D}$, which amounts to drop all Feynman graphs which do not contain at least one $\Phi^a(\hat{\nabla}^\alpha E^\alpha)^a$ vertex.

For the particular choice of purely transversal fields, $\partial^i\hat{A}^i = 0$, the operator sandwiched between the covariant derivative and the electric field becomes the spatial transversal projection operator, $\epsilon^i_\alpha\epsilon^j_\alpha = \delta^{ij} - \partial^i\partial^j/\partial^2 = T^{ij}(\partial)$. In that case, the $\Phi\nabla E$ terms in (8) turn into a single three-vertex $\Phi\hat{A}\hat{E}$ and not two-vertex $\Phi\hat{E}$ remains. We observe that even for a non-transversal choice of $\hat{A}, \hat{E}$, the perturbative $\hat{E}^L\hat{E}^L$ propagator is compensated by the $\hat{E}^L\Phi$ two-vertex and the $\Phi\Phi$ Green function. This corresponds to the fact that in pure QCD

zeroth order longitudinal states are pure gauge degrees of freedoms. We want to point out that although all fields are purely transversal they do not coincide with the transversal components of the original gauge potentials, and the transversal choice must not be confused with the Coulomb gauge.

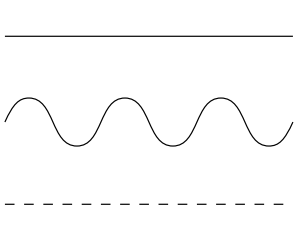
Splitting the Hamiltonian into a free part

$$\mathcal{H}_{\prime} = \frac{1}{2}\left(E^{a,i}T^{ij}E^{a,i} - A^{i,a}\triangle T^{ij}A^{i,a} + \Phi^{a}\triangle\Phi^{a}\right)$$

and an interacting one

$$\begin{aligned}
\mathcal{H}_{WW} &= \mathcal{H}^{\Phi AE} + \mathcal{H}^{\Phi A} + \mathcal{H}^{V(A)},\\
\mathcal{H}^{\Phi AE} &= gf^{abc}\Phi^{a}E^{Ti,b}A^{Ti,c},\\
\mathcal{H}^{\Phi A} &= gf^{abc}\Phi^{a}A^{Ti,c}\partial^{i}\Phi^{b} - \frac{g^2}{2}f^{abc}f^{ade}\Phi^{b}A^{Ti,c}A^{Tj,e}\frac{\partial^{i}\partial^{j}}{\partial^2}\Phi^{d},\\
\mathcal{H}^{V(A)} &= -gf^{abc}\partial^{j}A^{Ti,a}A^{Tj,b}A^{Ti,c} + \frac{g^2}{4}f^{abc}f^{ade}A^{Ti,b}A^{Tj,c}A^{Ti,d}A^{Tj,e},
\end{aligned}$$

the former one gives rise to the propagators



$$\begin{aligned}
\text{——————} &= \left\langle A^{Ti}A^{Tj}\right\rangle(p_0,\vec{p}) = -T^{ij}(\vec{p})\frac{1}{p_0^2-\vec{p}^{\,2}}\\
\text{∿∿∿∿∿} &= \left\langle E^{Ti}E^{Tj}\right\rangle(p_0,\vec{p}) = -T^{ij}(\vec{p})\frac{p^2}{p_0^2-\vec{p}^{\,2}}\\
\text{- - - - - - - - - - - - - - -} &= \quad \langle\Phi\Phi\rangle(p_0,\vec{p}) \quad = -\frac{1}{\vec{p}^{\,2}},
\end{aligned}$$

and from the second one we shall only need the expression for the $\Phi^{a}A^{Ti,b}E^{Tj,c}$ vertex given by $(-1)gf^{abc}\delta^{ij}$.

For two reasons, we do not express the electric fields in terms of time derivatives of the gauge potentials. Firstly, this would again involve inverting non-local operators, and subsequently render the theory untractable. Secondly, the kinetic term and the potential term are both by themselves physical observables. Keeping the E fields, one preserves the possibility to calculate the electric and magnetic dispersion independently.

At the present stage, the propagator of the field Φ only serves to write the inverse Laplacian in a way convenient for calculating quantities in ITF and remains unheated. In

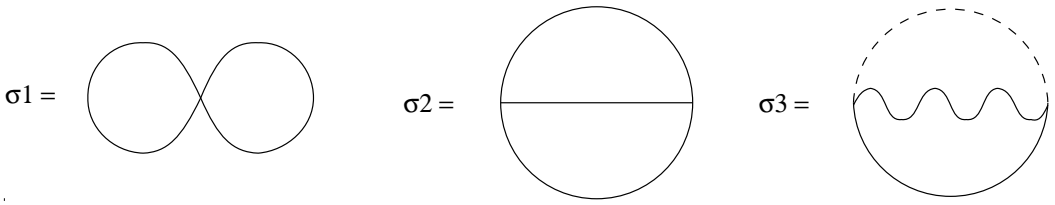


FIG. 1. Graphs contributing to the two-loop pressure in the physical subspace. Full lines correspond to $\hat{A}$ propagators, the wavy line to the $\hat{E}$-field propagator and the broken line to the auxiliary field.

the due course of an eventual resummation, however, consistency may require to assign the auxiliary field a temperature dependent Green function. This does, of course, not contradict the original nature of an auxiliary field, since to a given order, the perturbative inversion of $\mathcal{D}$ in the non-local Hamiltonian is not unique.

III. THE TWO-LOOP PRESSURE

In order to compare our method with the construction of James and Landshoff [16], we calculate the two-loop pressure given by the diagrams depicted in Fig1. We observe that apart from the third diagram, only transversal fields contribute to the pressure.

The temperature dependent contribution reads ($C_N = N(N^2-1)$ for SU(N))

$$Z^{(2)} = -g^2 C_N \frac{V}{T} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} \hat{Z}, \tag{9}$$

where $\hat{Z} = \sigma_1 + \sigma_2 + \sigma_3$ and σ_i corresponds to the three graphs in Fig1 respectively,

$$\sigma_1 = \frac{1}{4} \frac{(3-z^2)}{pq} \left(n_p n_q + n_p\right), \qquad \sigma_2 = \frac{1}{2} \frac{z\,(1+z^2)}{(\vec{p}+\vec{q})^2} \left(n_p n_q + n_p\right),$$

$$\sigma_3 = \frac{1}{4} \frac{1}{pq} \frac{(1+z^2)}{(\vec{p}+\vec{q})^2} (p^2+q^2) \left(n_p n_q + n_p\right),$$

$z = \vec{p}\vec{q}/pq,\ p = |\vec{p}|,\ n_p = (\exp(p/T)-1)^{-1}$, which gives the correct answer

$$P^{(2)} = \frac{T}{V} Z^{(2)} = -\frac{g^2 C_N T^4}{144}.$$

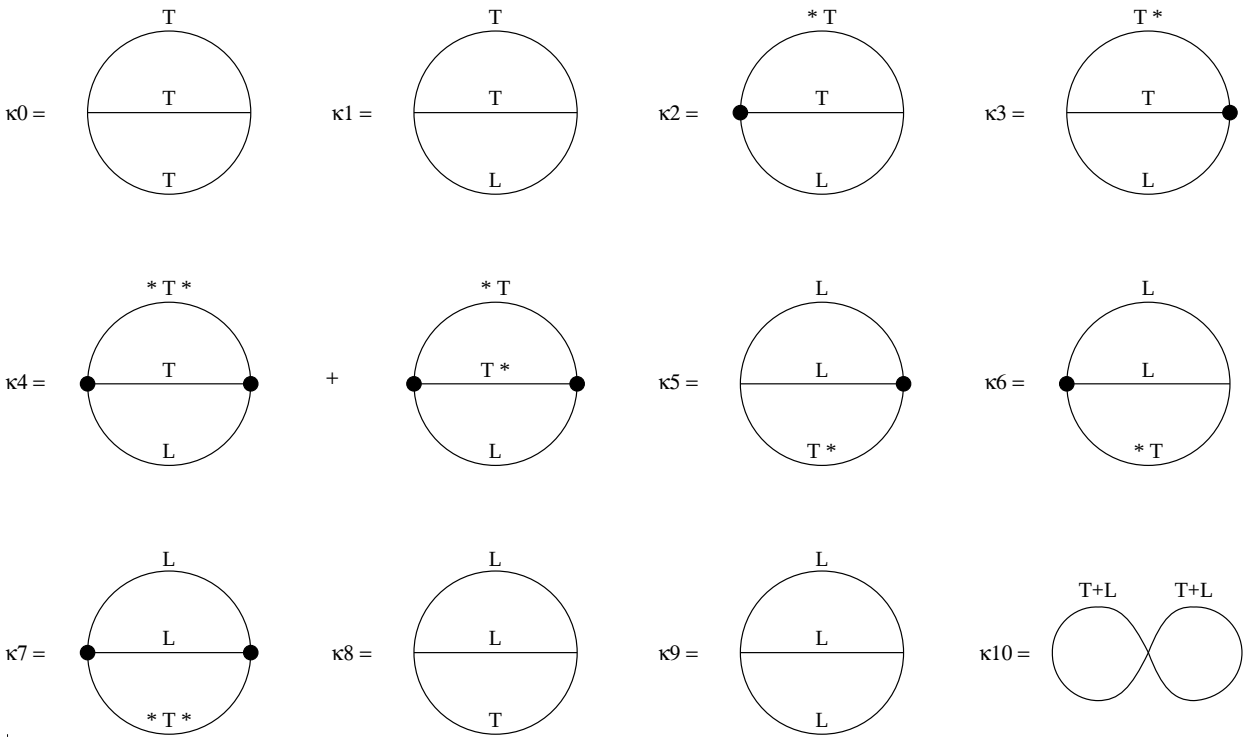


FIG. 2. Graphs contributing to the two-loop pressure in the approach of James and Landshoff. The blob denotes an additional three-vertex due to the explicit construction of Gaus law states.

Note the simplicity of the calculation. As opposed to that, the approach of James and Landshoff involves zero-temperature longitudinal propagators and heated transversal ones. The calculation of the pressure calls for the calculation of 12 graphs depicted in Fig2.

In addition to the usual three- and four-vertices of QCD, a static vertex denoted as a blob in Fig2 enters from the explicit construction of physical states satisfying the Gauss law constraint (4). The (TTL) part of that vertex is asymmetric in the transversal legs which we indicated by an asterisk. We neither want to repeat the details of the formalism nor the lengthy but straightforward calculation but rather state the result. Using the notation of Eq. (9) we find $\hat{Z} = \sum_{i=0}^{10} \kappa_i$ where (The contributions $\kappa_1 \ldots \kappa_4$ have already been calculated in [16].)

$$\kappa_1 = \frac{1}{2}\,\frac{1}{pq}\,\frac{(1+z^2)}{(\vec{p}+\vec{q})^2}\left[(n_p n_q + n_p)(p^2+q^2) + (2n_p+1)\frac{3T\,p^2 q}{p^2+q^2}\right],$$

$$\kappa_5 = \frac{p}{8T}\,\frac{(1-z^2)}{(\vec{p}+\vec{q})^2}(n_p + \frac{1}{2}),$$

$\kappa_0 = \sigma_2,\ \kappa_2 = -\frac{1}{2}\kappa_1,\ \kappa_3 = \kappa_2,\ \kappa_4 = \sigma_3,\ \kappa_7 = \kappa_6 = \kappa_5,\ \kappa_8 = -\kappa_5,\ \kappa_9 = 0,\ \kappa_{10} = \sigma_1$. All

(TLL) graphs cancel. Furthermore, since $\kappa_1 + \kappa_2 + \kappa_3 = 0$ only the term κ_4 remains in the (TTL) contributions that coincides with the third graph in Fig1 in our calculation. It is interesting to observe that only the (TTL) part of James and Landshoff's new vertex plays a rôle and that the only non–vanishing contribution comes from diagrams which contain a pair of that vertex part.

In the case of the two-loop pressure, one may even replace the longitudinal E-fields in the original Hamiltonian by the vertex part of the Gauss law constraint, i.e. $\vec{\partial}\vec{E} \to g f^{abc} E^{b,i} A^{c,i}$, and drop the longitudinal A fields to get the correct result. However, it is clear from our investigation that this oversimplifying guess does not give the correct answer in general.

IV. CONSTRUCTING GAUSS LAW STATES TO ARBITRARY ORDER

There are two different ways to handle gauge degrees of freedom at finite temperature. Based on BRS invariance, Hata and Kugo [23] constructed a theory where the Boltzman factor in the thermal average gets replaced by $\exp(i\pi N_c - H/T)$, N_c being the ghost number operator, and the trace is expanded to include ghost fields and all degrees of freedom of the gauge potential. They demonstrated that with this weight thermal averages of operators corresponding to observables are the same as in the projected ensemble. The advantage of their approach, which is the standard way finite temperature field theory is handled, is that all degrees of freedom are heated which results in simple Feynman rules. However, in the particular case of TAG, this construction does not work [5] and one has to go back to the projected ensemble involving physical states only.

There, the thermal average of an observable Q is defined by

$$\langle Q \rangle = Z^{-1} \sum \langle \mathrm{Phys} | \, e^{-H/T} Q \, | \mathrm{Phys} \rangle$$

where physical states satisfy the Gauss law, $\mathcal{G} \, |\mathrm{Phys}\rangle = 0$. In the formalism of James and Landshoff those are constructed by acting with a unitary operator on the free transversal states $|T\rangle$,

$$|\mathrm{Phys}\rangle = R\,|T\rangle\,, \quad R = \sum_{n=0}^{\infty} g^n R_n \tag{10}$$

where R_n was determined by acting $n+1$ times with the Gauss law operator on the physical states. There the longitudinal components of the gauge potential correspond to gauge degrees of freedom. In our approach, the decomposition (5) allows us to identify the longitudinal components with the 'angle' variable, $A^L = \partial\omega$, whereas the remaining field components read

$$A^T = A - A^L = e^{ig\omega}\hat{A}e^{-ig\omega} - \frac{i}{g}\left(\partial e^{ig\omega}\right)e^{-ig\omega} - \partial\omega. \tag{11}$$

Recalling that the field $\hat{A}$ contains but physical degrees of freedom, one realizes that the transversal components of A are physical only to zeroth order in g. Conversely, the choice to keep the longitudinal components A^L unheated to all orders corresponds to the fact, that the physical field $\hat{A}$ is transversal only to lowest order, which in turn means that there do appear higher order heated longitudinal modes contained in the states that satisfy the Gauss law.

The operator $\hat{A}$ can be expressed in (A^T, A^L) coordinates by virtue of (11)

$$\hat{A} = \sum_{n=0}^{\infty} g^n \hat{A}_n = e^{-ig\omega}\left(A^T + (\partial\omega) + \frac{i}{g}\partial\right)e^{ig\omega} = \sum_{n=0}^{\infty}\frac{(ig)^n}{n!}\overset{(n)}{[}\,\omega, A^T + \frac{n}{n+1}\partial\omega] \tag{12}$$

where $\overset{(n)}{[}\,X,Y] = [X, \overset{(n-1)}{[}\,X,Y]]$, $\overset{(0)}{[}\,X,Y] = Y$ denotes the multiple commutator. $\hat{A}$ is the counterpart of the Gauss law operator to arbitrary order, where the first few terms read

$$\hat{A} = A^T + ig[\omega, A^T + \frac{1}{2}\partial\omega] - \frac{g^2}{2}[\omega, [\omega, A^T + \frac{2}{3}\partial\omega]] + \ldots.$$

Since the change of the basis from physical to transversal states is mediated by a unitary transformation, the corresponding operators $\hat{A}$ and A^T are unitary equivalent according to $\hat{A}R = RA^T$. Collecting terms by orders of g, this leads to the recursion relation

$$[\hat{A}_0, R_n] + \sum_{m=0}^{n-1}\hat{A}_{n-m}R_m = 0.$$

The strategy to solve this recursion may be motivated by the following observation. R_n is given by the action of an (unknown) operator on the sum which inverts the commutator with

$\hat{A}_0 = A^T$. Recalling the equal time commutator $[A^{Ti,a}(x), E^{Tj,b}(y)] = i\delta^{ab}T^{ij}(\partial)\delta(x-y)$, one may, roughly speaking, construct R_n by 'multiplying' the sum with E^T.

In particular, for $n = 1$ one has to study the equation $[\hat{A}_0, R_1] + \hat{A}_1 = 0$ that has the solution

$$R_1 = i\,(E^T \cdot \hat{A}_1) := i \int d^3z \vec{E}^{T,a}(z)\hat{\vec{A}}_1^a(z).$$

Plugging in the expression for $\hat{A}_1$, the explicit form of R_1 can be written as ($\omega = \partial A^L/\partial^2$)

$$R_1 = if^{abc}\int d^3z \left(\frac{1}{\partial^2}\partial A^{L,a}\right)(z)\,\vec{E}^{T,b}\left(\vec{A}^{T,c}(z) + \frac{1}{2}\vec{A}^{L,c}(z)\right)$$

which coincides with the result of James and Landshoff.

The recursion for $n = 2$ reads

$$[\hat{A}_0, R_2] + \hat{A}_1 R_1 + \hat{A}_2 = 0. \tag{13}$$

Guided by what we have learned above, one would naively guess $R_2^{guess} = i\,(E^T\cdot\hat{A}_2) + {}^1\!/_2 R_1 R_1$ which when commutated with A_0 gives $[\hat{A}_0, R_2^{guess}] = -\hat{A}_2 - {}^1\!/_2\hat{A}_1 R_1 - {}^1\!/_2 R_1\hat{A}_1$ that cancels the third but not the second term in the recursion (13) since $\hat{A}_1$ does not commute with R_1. We therefore add a suitable chosen term proportional to the commutator which compensates the wrong order in the $R_1\hat{A}_1$ contribution. The solution of (13) reads

$$R_2 = i\,(E^T\cdot\hat{A}_2) + \frac{1}{2}R_1R_1 - \frac{i}{2}(E^T\cdot[R_1,\hat{A}_1])$$

which is unique up to operators that commute with $\hat{A}_0$.

One may continue further and calculate the $n = 3$ contribution to R,

$$R_3 = \frac{1}{6}R_1^3 + \frac{i}{3}\left(E^T\cdot\overset{(2)}{[}R_1,\hat{A}_1]\right) - \frac{i}{2}R_1\left(E^T\cdot[R_1,\hat{A}_1]\right) + iR_1\left(E^T\cdot\hat{A}_2\right) - \\ -i\left(E^T\cdot[R_1,\hat{A}_2]\right) + i\left(E^T\cdot\hat{A}_3\right),$$

and a calculation of higher order terms proceeds analog similar lines. We note that our construction has formal similarity with the Foldy-Wouthuysen transformation in quantum-mechanics.

Our result does not match the R_2 contributions found in [22] that also contain time derivatives of the longitudinal fields, but agrees with the argument given by James and Landshoff that those time derivatives should not be present in R. We also find that the exponentiation conjectured in [22] cannot be confirmed. Although the R_1 terms appear with the correct factors, a complete exponentiation is spoiled by an increasing number of commutator terms, which is consistent with the nonlocal Hamiltonian (7) that also contains an infinite number of vertices. Unlike as in pure QED, where the radiation gauge eliminates all gauge freedoms from the Hamiltonian in a local manner, and where R does exponentiate, is has been argued [20] that in nonabelian gauge theory no such canonical gauge exists.

V. CONCLUSION AND OUTLOOK

We formulated a finite temperature framework for pure QCD which is based on the elimination of all gauge degrees of freedom. In contrast to the former approach of James and Landshoff, who explicitly constructed physical states by solving the Gauss law constraint, we eliminate spurious degrees of freedom at the operator level which allows to heat the remaining degrees of freedom in a straight-forward way. We do not encounter any pole ambiguities which exist in the naive imaginary time formulation of axial gauges.

Although our effective Hamiltonian contains a kinetic term non-local in the fields, it is possible to find a local formulation by introducing an auxiliary field. We compared our theory with the construction of James and Landshoff for the particular case of the two-loop pressure and found that the number of Feynman graphs is reduced drastically in our framework. Since our construction allows to make a clear distinction between physical and gauge degrees of freedom, we can also line out a strategy of how to explicitly construct Gauss law states to all orders. Those are calculated explicitly up to third order in the basis of the free transversal states of the gauge field.

Clearly, we will establish a resummation program in our formalism. Since the Hamiltonian only contains physical observables, the location of the poles in the propagators contains

intrinsic physical information on the the on-shell dispersion relation. It would be interesting to compare with results obtained in the usual approach, where gauge-fixing independence of the poles has to be and was proven [24].

Furthermore, we only dealt with pure QCD, which by construction excludes the calculation of the Debye-mass that would require the gauge-invariant inclusion of a charge density in the Gauss law constraint. Conversely, if it turned out to be possible to include charges in the present formalism it would be possible to separate effects from external charges and those induced by pure QCD. We are going to investigate on this question.

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